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Resource Discoveries, Learning, and National Income Accounting

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Abstract

Questions about the ultimate size of mineral and energy resource endowments and the degree of fiscal prudence which should be exercised by countries engaged in resource extraction have become central for many developing countries during the recent resource boom. To explore these questions, this paper develops a model of optimal resource extraction and discovery that combines two polar assumptions: (i) that discovering a resource today drives up the cost of future resource discoveries, and (ii) that extracting resources yields knowledge that reduces the cost of discovery. Although the model shows that resource discoveries should be valued at marginal discovery cost in measures of national saving and income, the ultimate size of the resource that can be exploited is the result of the interplay between rising discovery costs and accumulating knowledge. Empirical tests of the model show that the resulting income estimates would be extremely volatile for many extractive economies, owing to the lumpiness of resource discoveries. Two alternative accounting approaches, based on Hicksian concepts, yield more intuitive and less volatile income estimates. The question of fiscal prudence for extractive economies hinges on how optimistic countries are about the risks in future mineral and energy markets, and how far into the future these countries are willing to project optimistic trends when making decisions about how much to consume and how much to save of current resource revenues.

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Resource Discoveries, Learning, and National Income Accounting

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Resource Discoveries, Learning, and National Income Accounting

Introduction

Depletion of subsoil assets such as petroleum and minerals amounted to more than 10% of GNI in 25 countries in 2009 according to the *World Development Indicators* (World Bank 2011a). Subsoil resources are not only depleted, however. These resources are also discovered as a result of exploration activities. The deterministic model of resource extraction and discovery of Pindyck (1978) was the forerunner of a series of papers dealing with the theory of resource discoveries, covering issues such as heterogeneous resource quality and the role of uncertainty.[2]

Resource depletion and discovery have implications for the measurement of national income and the sustainability of the economies where these activities take place. Following Weitzman's (1976) growth-theoretic exposition of national accounting, the treatment of resource depletion and discovery in national income has been analyzed by Hartwick (1993) and appears as one of a series of special cases in Arrow *et al*. (2003). Our goal here is to develop a model of resource discovery with learning, building on Arrow *et al*. (2003), and to combine this with the key characteristic of the Pindyck (1978) model, the assumption that cumulative discoveries drive up discovery costs. We develop measures of income, saving, and wealth for this model, and derive the basic dynamics. Since it is possible for consumption to approach 0 asymptotically under optimal depletion and discovery, we also derive a rule for sustainability in the model using a generalization of the Hartwick Rule (Hartwick 1977). We then proceed to use this model to develop empirical measures of income and saving.

Key characteristics of our model include: (i) the ultimate undiscovered stock of the resource is finite; (ii) resource discovery essentially transfers a quantity of the ultimate stock from the 'undiscovered' category to the 'discovered' category; (iii) discovery costs increase as cumulative discoveries rise, potentially offsetting the effects of learning; and (iv) there is learning, in the form of resource discovery costs that decrease as cumulative extraction of the resource increases. Learning therefore introduces endogenous technological progress in the discovery cost function.[3]

As estimates presented in Gelb *et al*. (2012) show, an important issue for income measurement in exhaustible resource economies is the lumpiness of resource discoveries, which often equal 100% or more of GDP. A strictly 'marginalist' approach to national accounting with discoveries, which would value the *total* discovery as part of income in the year it is made, would therefore lead to extreme volatility of measured income – see, for instance, Repetto *et al.* (1989). We develop alternatives to the marginalist approach for measuring income and saving which are motivated by the theoretical model, using a modified Hicksian income framework for discoveries. We present empirical applications of these alternative approaches and conclude with thoughts on the use of wealth accounting to guide prudent fiscal policies in extractive economies.

[2] A partial list of papers includes Cairns and Quyen (1998), Devarajan and Fisher (1982), Deshmukh and Pliska (1980), Lasserre (1984), and Swierzbinski and Mendelsohn (1989).
[3] Farzin (2001) focuses on additions to existing reserves and assumes exogenous technological change in the exploration cost function.

Definitions and model assumptions

We define the ultimate undiscovered resource stock X, and resource discovery cost function $v(D, Z, Q)$ which depends on current discoveries D, cumulative extraction Z, and cumulative discoveries Q. The production function is $F(K, R)$ for produced capital K and resource extraction R, the utility function is $U(C)$ for consumption C, and there is a constant pure rate of time preference ρ. The current stock of discovered – or 'proven' – resources available for extraction, S, is purely a function of the other variables and so may be eliminated from the model.[4] All variables are assumed to be functions of time unless otherwise specified.

Social welfare V is equal to the present value of utility:

$$V = \int_t^\infty U e^{-\rho(s-t)} ds \tag{1}$$

The cost of resource discovery is an increasing function of the amount discovered and of cumulative discoveries,

$$v_D > 0, \ v_Q > 0 \tag{2}$$

and a decreasing function of cumulative extraction,

$$v_Z < 0. \tag{3}$$

We also assume decreasing marginal utility of consumption,

$$U_{CC} < 0 \tag{4}$$

decreasing marginal returns to production factors, quasi-complementarity, and constant returns to scale (CRS),

$$F_{KK} < 0, \ F_{RR} < 0, \ F_{KR} > 0, \ F_{RK} > 0, \ F(K, R) = F_K K + F_R R. \tag{5}$$

As we will see, extraction may or may not be efficient in the sense that all resources (discovered and undiscovered) are fully exploited over an infinite time horizon.

The model of resource extraction and discovery

The objective of this simple economy is to maximize social welfare over an infinite time horizon. Along the development path the accumulation equations are given by

$$\dot{K} = F - C - v \tag{6}$$

$$\dot{Z} = R \tag{7}$$

$$\dot{X} = -D \tag{8}$$

[4] Specifically, $S(t) = X(0) - X(t) - Z(t)$.

$$\dot{Q} = D \tag{9}$$

and the Hamiltonian for the optimization problem for shadow prices of stocks γ_i is given by

$$H = U + \gamma_K \dot{K} + \gamma_Z \dot{Z} + \gamma_X \dot{X} + \gamma_Q \dot{Q} \tag{10}$$

The static first-order conditions for an optimum therefore yield

$$\gamma_K = U_C$$

$$\gamma_Z = -U_C F_R$$

$$\gamma_Q - \gamma_X = U_C v_D$$

For this economy, therefore, the expressions for adjusted net (genuine) saving and adjusted net national income are

$$ANS = \dot{K} - F_R R + v_D D \tag{11}$$

$$aNNI = C + \dot{K} - F_R R + v_D D \tag{12}$$

Note that $v_D D$ is the value of resource discoveries, which is distinct from the cost of resource discoveries v. In order to understand the dynamics of saving and income on the optimal path, we need to derive the dynamic first order conditions for the shadow prices, which yields

$$\frac{\dot{U}_C}{U_C} = \rho - F_K \tag{13}$$

$$\frac{\dot{F}_R}{F_R} = F_K - \frac{v_Z}{F_R} \tag{14}$$

$$\frac{\dot{v}_D}{v_D} = F_K + \frac{v_Q}{v_D} \tag{15}$$

Expression (13) is just the Ramsey formula, while expressions (14) and (15) are modified Hotelling rules. Resource rents rise faster than the interest rate, and so do marginal discovery costs.

Defining $\omega = -v_Z$ to be, loosely speaking, the value of knowledge about resource deposits, solving expression (14) yields the result that the instantaneous value of the scarcity rent at time *t* just equals the accumulated value of knowledge, including interest, plus the initial value of the scarcity rent at time 0,

$$F_R = \int_0^t \omega(s) e^{\int_s^t F_K(\tau) d\tau} ds + F_R(0) \tag{16}$$

Substituting expression (14) into expression (15) yields,

$$\frac{\dot{v}_D}{v_D} - \frac{\dot{F}_R}{F_R} = \frac{v_Z}{F_R} + \frac{v_Q}{v_D} \tag{17}$$

In the Pindyck model the term v_Z/F_R in expression (17) is 0 – there is no learning. As a result, marginal discovery costs rise faster than resource rents. Resource discovery ceases to be profitable when marginal discovery costs exceed resource rents and, depending on functional forms and initial conditions, this point may be reached before the stock X is fully discovered. We can think of this as the point when 'profitable discoveries are exhausted.'

In a pure learning model with no effect of cumulative discoveries on resource discovery costs, the term v_Q/v_D in expression (17) is 0. Assuming that resource discovery is profitable at the start of the program, so $F_R(0) > v_D(0)$, it follows that resource rents will be larger than marginal discovery costs along the optimal path for all $t \geq 0$, and as a result the stock X will be fully discovered over the infinite time horizon.

Expression (17) therefore captures the mixed nature of our model. Depending on initial conditions and functional forms, resource discovery may or may not cease to be profitable at some finite point in time, depending on the relative strengths of the effects of learning and the exhaustion of profitable discoveries. The undiscovered resource stock X may or may not be fully discovered.

Note that efficient resource extraction from a finite endowment impies that both R and D must eventually fall asymptotically to 0, and so expressions (2) and (3) imply that discovery costs must also eventually decline, in part because the cumulative stocks Q and Z must plateau at maximum values less than $S + X$.

The final point to note on dynamics is the effect of diminishing marginal returns to factors of production. Since resource extraction R must eventually fall asymptotically to 0 and resources are essential for production, it follows that capital must steadily accumulate in order to sustain production and consumption. Assuming that the marginal product of capital F_K has no positive lower bound, expression (5) implies that F_K will eventually fall below the pure rate of time preference ρ, at which point the Ramsey rule (expression 13) will drive consumption asymptotically to 0 on the optimal path (cf. Dasgupta and Heal 1979). Under the assumptions given, the optimal resource extracting and discovering economy is unsustainable, and this motivates our discussion of sustainability below.

Saving, income, wealth and sustainability

If we denote adjusted net saving (expression 11) as G (genuine saving), then the Annex derives the two key characteristics of the saving measure:

$$G = \frac{1}{U_C}\dot{V} \tag{18}$$

$$\dot{C} = F_K G - \dot{G} \tag{19}$$

The first expression indicates the welfare significance of genuine saving. If the objective is to maximize social welfare for a fixed pure rate of time preference, then genuine saving G equals the dollar-valued change in social welfare at each point in time.[5]

While expression (18) depends upon the assumption of optimality, expression (19) requires only that assets be priced efficiently, as in expressions (14) and (15). If this holds, then consumption will be increasing as long as genuine saving is positive and growing at less than the rate of interest – this is the generalized Hartwick Rule (see Hartwick 1977, Dixit *et al.* 1980, Hamilton and Hartwick 2005, Hamilton *et al.* 2006, and Hamilton and Withagen 2007).

As seen in expression (12), the measure of adjusted net national income (*aNNI*) for this economy is given by,[6]

$$aNNI = C + G \tag{20}$$

For the optimal economy, it follows immediately from expressions (18) and (19) that growth in *aNNI* is proportional to the change in social welfare (cf. Asheim and Weitzman 2001 and Hamilton and Ruta 2009),

$$\dot{C} + \dot{G} = \frac{F_K}{U_C}\dot{V} \tag{21}$$

We can also derive an expression for the growth rate of *aNNI* under the generalized Hartwick Rule using expression (19),

$$\frac{\dot{C}+\dot{G}}{C+G} = \frac{F_K G}{C+G} = \frac{F_K}{1+\frac{C}{G}} \tag{22}$$

The growth rate of *aNNI* increases with genuine saving G.[7] If the return on capital F_K is 9% and consumption C is twice as large as genuine saving G, then the growth rate of *aNNI* will be 3%. Expression (22) is a useful result because it says that a policy for sustaining development[8] is also a policy which leads to income growth.

To aid intuition, it is worth considering a specific instance of the generalized Hartwick rule. If we assume a *constant* value of genuine saving $\sigma > 0$, then the policy rule for sustaining development in the current model is given by,

$$G = \dot{K} - F_R R + v_D D = \sigma \quad \forall t \tag{23}$$

As long as total investment in produced capital plus the value of resource discovery is greater than resource depletion by a fixed amount σ, then expression (19) implies that consumption increases.[9] The case $G = \sigma = 0 \ \forall t$ is just the classic Hartwick Rule, which leads to constant consumption.

[5] See Hamilton and Clemens (1999). This expression also holds in non-optimal economies for suitable definition of the shadow prices, as shown in Dasgupta and Mäler (2000).
[6] For a fuller discussion of alternative income measures, see Asheim (2000).
[7] Hamilton and Withagen (2007) derive the equivalent expression for an economy with a Cobb-Douglas production function.
[8] i.e. non-decreasing social welfare at each point in time.
[9] For a fuller elaboration of constant genuine saving in a Dasgupta-Heal economy, see Hamilton *et al.* (2006).

Finally, we derive the wealth accounts under the assumption of constant returns to scale (CRS). Denote the value of total resources as M and define tangible wealth Q as follows,

$$Q \equiv K + \int_t^{\infty} F_R R e^{-\int_t^s F_K(\tau)d\tau} ds \equiv K + M$$

Here the path for resource extraction R exhausts the total resource endowment – proven reserves S plus the quantity of resource which can be discovered profitably. Now CRS and expression (6) lead to the following derivation,

$$\begin{aligned} \dot{Q} &= \dot{K} + F_K M - F_R R \\ &= F_K K + F_R R - C - v + F_K M - F_R R \\ &= F_K Q - (C + v) \end{aligned}$$

This equation has particular solution,

$$K + M = \int_t^{\infty} (C + v) e^{-\int_t^s F_K(\tau)d\tau} ds$$

Since total wealth W is equal to the present value of consumption under constant returns to scale, this yields

$$W = \int_t^{\infty} C e^{-\int_t^s F_K(\tau)d\tau} ds = K + M - \int_t^{\infty} v e^{-\int_t^s F_K(\tau)d\tau} ds \tag{24}$$

This makes intuitive sense: total wealth is the sum of produced capital plus the value of the total resource endowment, minus the present value of the expenditures required to discover these resources.

Applying the theory to practical wealth accounting

An important aspect of formal models such as the one just presented is the guidance the model provides to questions of measurement. We begin this section by discussing the treatment of resource discoveries in national accounting, then turn to an alternative approach which abstracts from discoveries and focuses instead on national accounting based on the total expected value of natural resources stocks.

(i) Alternative ways to account for discoveries: a modified Hicksian approach

The theoretical model suggests in expressions (11) and (12) that measures of net income and saving should be increased by the quantity of resource discovered valued at the marginal discovery cost, $v_D D$. However, as seen in Gelb *et al.* (2012), and as we shall see in the empirical portion of this paper, resource discoveries tend to lumpy and large relative to GDP. Rather than treating the full value of the discovery as an addition to income and saving in the year the discovery is made, it seems more logical to adopt a Hicksian approach, whereby national income increases by the *return* on the increment to wealth, rather than by the full increment (Hicks 1946).[10]

[10] Asheim (2000) provides a careful analysis of the links between changes in wealth and alternative definitions of income.

We explore the Hicksian approach by assuming that a new resource deposit of value N has been discovered, where,

$$N = \int_t^T q(s)R(s)e^{-r(s-t)}ds \tag{25}$$

for unit rent q, extraction R, resource lifetime $T - t$, and constant discount rate r. It follows immediately that the Hicksian income derived from resource extraction is given by,

$$rN = \dot{N} + qR \tag{26}$$

Here qR is the total rent on extracting the resource. However there is a problem with this expression if the objective of accounting is to measure changes in social welfare. As Hamilton and Ruta (2009) show, in optimal economies there is no direct relationship between $\dot{N}$ (the total change in the value of the resource stock as a result of extracting quantity R, including capital gains) and the change in social welfare.

Hamilton and Ruta (2009), building on Dasgupta and Mäler (2000), establish how to measure the marginal change in real wealth[11] when a unit of resource is depleted in non-optimal economies. Under reasonably general conditions the unit value of depletion n is,

$$n \equiv \frac{\partial W}{\partial S} = \frac{\partial N}{\partial S} = \frac{N}{S} \tag{27}$$

for total wealth W, resource stock value N and physical resource stock S.[12]

Multiplying n by the quantity of resource extracted R gives a measure of resource depletion, and this will be less than the total rent on extraction. As a consequence, total rent may be partitioned into a depletion portion and an income portion. Using expression (27), Figure 1 plots the share of depletion in total rents as a function of the size of the resource stock, assuming a discount rate r of 4%, and constant unit rents and quantity of resource extracted in each year.[13] As expected, the depletion share of total rent falls with the size of the resource deposit. At a resource lifetime of 100 years the depletion share is 25.5%, and this increases steadily to 100% at the point of exhaustion.

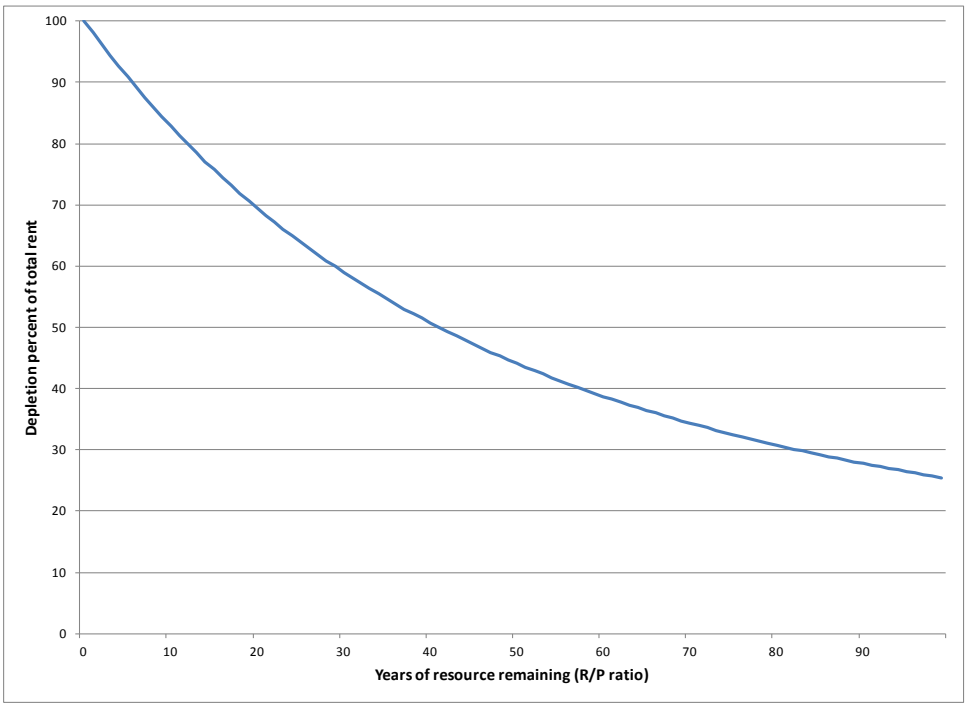
[11] Hamilton and Ruta (2009) derive the real change in wealth when a marginal unit of resource is extracted, under the assumption of constant unit rent and constant quantity of resource extracted. Their unit value of the change in real wealth – the accounting price – corresponds to n in expression (27). They then show that the change in total resource wealth when the resource is extracted is given by $\dot{N} = n\dot{S} + \dot{n}S$. It follows that $\dot{N}$ consists of the change in real wealth plus a capital gains term. For a closed economy, of course, capital gains are just transfers and have no impact on aggregate social welfare. For open economies, the case of small resource exporters is discussed below under the heading 'Empirical application.'

[12] Expression (27) is not a general result, but it is straightforward to show that it holds if the unit rent q is independent of extraction R. This would be the case, for example, if the country is a price taker on world markets and marginal extraction costs are constant.

[13] If the assumptions of constant extraction and constant unit rents are relaxed, then it may be necessary to derive the accounting price n numerically. In this case the precise values shown in Figure 1 will change, but the general trend in depletion as a share of total rent will continue to hold. In particular, the share of depletion in total rent will approach 100% as the resource lifetime approaches 0.

Because the total rent on resource extraction is part of GNI, the effect of resource discovery is to increase net income, *aNNI*, by decreasing the depletion share of total rent in the years subsequent to the discovery.

Figure 1. Depletion share of total rent vs. resource lifetime, *r* = 4%



Source: authors

By using the schedule of resource depletion shares shown in Figure 1 for a 4% discount rate, we can construct an example of how a large resource discovery would affect the net national income of a country. Assume that a country has 10 years of reserves and that total rent on extraction (assumed to be constant) is equal to 40% of *aNNI* in the base year. Now assume that the country discovers an additional 15 years of resource reserves, bringing the total to 25 years. Before the resource discovery the depletion share of total rent was 84.4%, while after the discovery this drops to 65.0%. Given the assumed share of rents in *aNNI* and the calculated drop in the depletion share, the net effect of the resource discovery is therefore to increase *aNNI* by about 8%. In contrast, the purely Hicksian approach to discoveries (expression 26) would increase income by roughly 12% – this would be an incorrect measure of income because it includes capital gains.

To make the accounting approach explicit, assume that a resource discovery of quantity D is made in year t. Assuming that the quantity extracted remains at a constant level R up to the point of exhaustion of the new larger stock, we can define the new stock as $S^* = S + D$ and the new value of this stock as $N^* > N$, while the pre-discovery unit value of depletion is given by n. Under the assumption of constant unit rent and constant extraction level R, Figure 1 indicates that the new unit value of depletion would satisfy,

$$n^* = \frac{N^*}{S^*} < n \tag{28}$$

Referring back to our expression for *aNNI*, expression (12), we therefore define the 'marginalist' value of national income as,

$$aNNI = C + \dot{K} + n(D - R) \tag{29}$$

While in the optimal economy we know that the marginal discovery value must be less or equal to the marginal rental value of a unit of resource (resource discovery is unprofitable otherwise), here we assume that the pre-discovery unit depletion value can be used to value both the quantity depleted and the quantity discovered – this is the approach taken in Gelb *et al.* (2012) as well as Repetto *et al.* (1989).

The logic of the 'modified Hicksian' or 'change in balance sheet' approach to measuring national income is that, rather than treating all of the resource discovery as income in the year of discovery (as shown in expression 29), the effect of the discovery is to boost income by reducing the unit depletion value n^*, so that the new measure of national income becomes,

$$aNNI^* = C + \dot{K} - n^* R \tag{30}$$

Obviously if no further resource discoveries are ever made, then for all years beyond year *t* the two measures of national income coincide and are given by expression (30).

(ii) Alternative ways to account for discoveries: the total expected resource approach

Expression (24) in the theoretical model suggests another approach to dealing with resource discoveries, an approach that completely abstracts from the discovery process. While in the theoretical model there is an implicit total resource extent (the sum of discovered plus undiscovered resources), in practice countries with Geological Survey institutions often publish estimates of probable resources based upon an assumed probability distribution. For these countries it is feasible to arrive at the total resource extent as the sum of proven reserves plus the expected value of probable resources. It then becomes possible to measure national income based upon the total wealth estimates given by expression (24)[14].

To make this approach explicit, assume that X^* measures the expected value of probable resources. Then the total resource stock is given by $S^* = S + X^*$, which leads to a revised estimate of the unit value of depletion n^* as shown in expression (28), which in turn leads to a measure of net national income as given by expression (30). This is obviously another 'modified Hicksian' approach.

Empirical application – making maximum use of geological information

Gelb *et al.* (2012) conclude their discussion of resource discovery by raising a fundamental question: what is a prudent fiscal policy for exhaustible resource extracting economies in the face of the likelihood of further resource reserves – the result of discovery expenditures – and uncertainty about the extent of these future reserves? At issue, of course, is the question of what proportion of resource revenues should be saved rather than consumed. Since the share of total rents that is not depletion is by definition available to be consumed, calculating

[14] This approach requires estimates of discovery costs as well, as seen in expression (24).

measures of income, saving and wealth can provide a hypothetical upper bound on what a prudent fiscal policy would look like.

First we look back at the recent resource discovery record that can be inferred from changes in proven reserves, or *S*, over a period of time. We then apply these discovery data to alternative measures of national income using the marginalist approach suggested by the theoretical model, and the modified Hicksian approach. Next we exploit data on total resources (proven reserves plus the expected value of probable resources) to examine how expanding the national balance sheet to reflect total resources would affect measures of depletion and net saving.

(i) Modified Hicksian approaches to discoveries compared to marginalist approaches

To compare modified Hicksian versus marginalist approaches for the calculation of *aNNI*, we first adjust investment ($\dot{K}$) by subtracting the depreciation of produced capital, calculate alternative measures of resource depletion using expressions (27) and (28), and then we apply expressions (29) and (30) in order to arrive at our alternative income measures.

Total resource rents are estimated using data on world resource prices and (average) extraction costs from World Bank (2011a).[15] The values of resource stocks are estimated as present values of resource rents using a social discount rate of 4%.

We use published data on proven reserves *S* for oil resources over the period 2000-2009 drawn from BP (2011). Following the usual definition, these proven reserves are stocks of a resource that can be exploited profitably at current prices and costs. As noted by Gelb *et al*. (2012), these data can be used to infer imputed discoveries in any given year. That is, for example, the physical difference between the (closing) stock in 2008 and the (closing) stock in 2009 (gross of extraction over the year) can be thought of as the implied amount of resources added to proven reserves over the year. Broadly speaking, these added resources might come from two sources (for a discussion, see Mitchell, 2004): (i) known resources and, (ii) unknown resources. In the former, additions to proven resources are from discovered sources which were contingent on, for example, technological improvement which would allow extraction to be economically feasible. This might entail a technological development that now allows more of an oil resource to be extracted from known wells. In the latter, these additions are from more speculative or 'undiscovered' sources where, for example, geological evidence suggests that resources are present with some probability. Of course, proven reserves might also be revised downwards. More generally, evolving knowledge will determine assessments about the allocation of resources between these categories as well as leading to revisions in estimated total resources.

Table A.1 in the Annex to this paper describes some basic data needed to calculate the implications of a marginalist approach to income measurement given the apparent extent of imputed discoveries over 2000-2009. As well as the physical extent of imputed discoveries, the table also describes this calculation as a percentage of proven reserves. For example, in the case of Ecuador in 2008, imputed discoveries corresponds to almost 68% of the closing stock of proven oil reserves. This information about imputed discoveries is also conveyed in terms of the number of implied years that are added to the reserve life (calculated as the

[15] Note that if world resource prices are distorted, as a result of monopoly or oligopoly for example, the values of resource rents will also diverge from efficient levels.

tonnes of imputed discovery divided by tonnes of production of oil in a given year). To use the example of Ecuador once more, in 2008, imputed oil discoveries of 368.9 million tonnes (mt) add just over 14 years to the reserve life for (proven) oil resources for that country.

As previously mentioned, imputed discoveries can be negative where, for example, there is a downward revision of proven reserves. This appears to have happened for Trinidad and Tobago in 2003 and 2004, Norway in 2006 and Ecuador in 2006, 2007 and 2009. Also notable are some of the relatively large spikes in imputed discoveries. For example, for Kazakhstan in 2001, imputed discoveries accounted for about 60% of end-year proven reserves. This very large upward revision had the effect of adding almost 55 years to Kazakhstan's oil reserve life. Imputed discoveries are similarly large for Venezuela in both 2008 and 2009 adding considerably to proven reserves and, by the same token, to reserve life.

Tables 1a & 1b illustrate our findings for the period 2000 to 2009 in terms of *aNNI* and its growth over the period. Table 1a presents the marginalist approach where discoveries are counted as income. The top panel in the table indicates the dollar value of *aNNI* in 2000 as well as the percentage growth rate in this income measure over the rest of the period. The bottom panel illustrates the contribution to overall growth in *aNNI* of changes in (discovery-adjusted) depletion values. For example, for Angola in 2001, the percentage change in depletion value of -39.8% means that *aNNI* in that year was about 39.8% lower, than in the year 2000, because of an increase in the value of (discovery-adjusted) oil depletion. These depletion values might change for a number of reasons including, for example, changing (real) resource prices as well as resource discoveries which occur over a year.[16]

For Table 1a, the data make it clear that, for the marginalist approach, the (real) dollar value of *aNNI* oscillates upwards and downwards within relatively wide bounds for most of the 10 countries illustrated in the table. Indeed, in a number of instances these changes are dramatic. An extreme case is that of Azerbaijan where in 2002 the estimate of *aNNI* shows an almost 20-fold upward spike in income compared the previous year. Crucially, almost all of that change was attributable to a large value of the change in (discovery-adjusted) depletion. The reason for this can be surmised from the data in Annex Table A1. Imputed discoveries for Azerbaijan in 2002 were 85% of end-year proven reserves. Taken at face value, the reserve data from 2003 onwards for Azerbaijan also indicate that imputed discoveries just matched oil production in each of these years. As a result, following the large upward spike in *aNNI* in 2002, measured net income drops precipitously (by over 90%) the following year.

While extreme, the experience of Azerbaijan is by no means exceptional when looking at *aNNI* through the lens of this marginalist approach. Kazakhstan shares a similar experience owing to a major (imputed) discovery adding to proven reserves one year earlier than in Azerbaijan. Single large upward spikes followed by large declines are evident in Ecuador and Sudan. Angola and Venezuela experience two such large upswings and downswings during the period 2000 to 2009. For those countries in Table 1a which do not experience these large oscillations, Annex Table A1 indicates that the explanation for this is relatively low and steady levels of imputed discoveries relative to proven reserves.

[16] Vincent *et al.* (1997) and Hamilton and Bolt (2004) show that if resource prices are exogenous and follow a consistent trend, then there will be capital gains on resource exports and imports which should be included in measures of saving and income. Here we assume that there is no long run price trend for crude petroleum, but it is worth noting the conflicting evidence (Hamilton 2008, Livernois 2009) that physical scarcity is starting to play a role in the upward trend in oil prices since 2000.

While there is some divergence in experience across these 10 countries, the extremely high volatility inherent in the marginalist approach raises the question that Hicks posed – whether unexpected windfalls should be treated as part of income and saving. As an alternative, Table 1b recalculates *aNNI* by valuing depletion according to the modified Hicksian approach. On the whole, the year-to-year changes in *aNNI* are smaller and far less of this change is driven by changes in depletion values, compared with the figures in Table 1a. [17] It is arguable that this presents us with more intuitively appealing estimates of an oil-producing country's (Hicksian) income.[18]

[17] There are exceptions, notably Angola and Azerbaijan. In the case of the former, for example, the large negative contributions of the change in depletion values to *aNNI* is explained by an increasing unit rent over the period 2003-2008 as well as increasing production. This unit rent then fell in 2009. A similar pattern emerges in the case of Azerbaijan.

[18] The modified Hicksian approach also accords well with the UN standard for the System of Environmental-Economic Accounting (United Nations 2012).

Table 1a: Estimates of Adjusted Net National Income Using a Marginalist Approach

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
	aNNI: $m, 2000	***Percentage growth in (real) aNNI per annum***								
Angola	$20,746	-40.7%	195.1%	-75.1%	100.4%	4.0%	41.5%	674.3%	-75.8%	-2.2%
Azerbaijan	$4,328	5.4%	1927.1%	-93.9%	13.7%	41.1%	56.4%	51.9%	54.1%	-7.7%
Ecuador	$14,823	21.6%	36.6%	-8.6%	11.0%	-8.7%	-20.6%	-7.5%	840.7%	-79.3%
Kazakhstan	$14,079	621.3%	-81.3%	22.2%	35.2%	32.4%	24.6%	25.2%	21.5%	-11.2%
Norway	$151,636	-2.7%	-4.2%	26.0%	10.3%	20.3%	-10.7%	27.8%	4.2%	-12.5%
Russian Federation	$237,601	35.4%	26.0%	-2.8%	23.5%	31.7%	-1.5%	50.8%	47.2%	-34.6%
Saudi Arabia	$171,079	-6.9%	3.5%	9.9%	21.3%	13.3%	11.0%	3.4%	20.1%	-17.9%
Sudan	$17,885	-29.4%	-5.8%	966.3%	-86.1%	19.7%	47.9%	10.8%	13.2%	-6.7%
Trinidad & Tobago	$7,142	19.8%	10.2%	-45.9%	63.4%	43.9%	6.3%	42.2%	-5.4%	-23.3%
Venezuela	$102,341	7.7%	-32.6%	-9.4%	67.6%	3.4%	115.4%	37.2%	319.6%	-69.5%
		Percentage contribution of (discovery-adjusted) depletion to (real) aNNI growth per annum								
Angola		-39.8%	179.0%	-80.3%	58.6%	-22.6%	0.0%	628.7%	-81.2%	0.0%
Azerbaijan		0.0%	1920.4%	-94.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ecuador		-8.1%	21.5%	-18.6%	0.0%	-20.2%	-32.6%	-17.7%	805.0%	-78.1%
Kazakhstan		599.6%	-83.1%	0.0%	0.0%	8.5%	-6.4%	0.0%	0.0%	0.0%
Norway		-3.2%	-14.4%	8.6%	-2.8%	4.7%	-19.0%	14.0%	-8.6%	9.3%
Russian Federation		18.7%	17.2%	-18.6%	-9.8%	5.9%	-18.3%	18.6%	12.4%	-8.6%
Saudi Arabia		-0.2%	0.7%	-0.7%	9.9%	-8.7%	0.9%	-0.6%	-1.1%	3.3%
Sudan		-33.3%	-13.4%	948.9%	-88.1%	-6.5%	21.9%	-7.0%	-6.9%	0.0%
Trinidad & Tobago		13.4%	9.1%	-62.4%	36.3%	19.0%	-1.3%	26.9%	-28.7%	12.4%
Venezuela		5.7%	-7.5%	2.5%	37.7%	-18.6%	93.2%	23.8%	303.7%	-69.8%

Source: authors' own calculations and adapted from BP (2011), World Bank (2011a), Gelb *et al*. (2012)

Table 1b: Estimates of Adjusted Net National Income Using a Modified Hicksian Approach

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
	aNNI: $m, 2000	***Percentage growth in (real) aNNI per annum***								
Angola	$2,758	29.0%	43.8%	25.0%	23.0%	-15.7%	46.8%	91.3%	-2.9%	82.7%
Azerbaijan	$2,474	20.3%	38.8%	16.5%	9.8%	13.8%	29.1%	49.7%	67.4%	10.3%
Ecuador	$10,935	43.6%	18.2%	10.9%	4.2%	5.8%	7.0%	6.4%	21.5%	4.3%
Kazakhstan	$12,345	29.5%	8.3%	20.7%	30.6%	19.7%	34.0%	27.1%	17.4%	-6.7%
Norway	$120,049	4.3%	12.1%	15.5%	10.3%	12.3%	7.7%	13.9%	9.0%	-13.7%
Russian Federation	$198,931	22.5%	10.9%	19.7%	34.1%	22.0%	16.4%	31.3%	36.1%	-26.4%
Saudi Arabia	$143,325	-3.7%	4.1%	5.5%	5.6%	16.8%	9.2%	5.3%	13.7%	-9.1%
Sudan	$8,916	8.8%	7.5%	26.5%	15.3%	26.6%	24.5%	14.4%	20.4%	-0.7%
Trinidad & Tobago	$5,776	11.6%	0.0%	20.2%	14.9%	14.6%	5.1%	20.5%	27.7%	-29.9%
Venezuela	$92,094	4.9%	-27.6%	-11.9%	26.3%	26.3%	26.5%	29.0%	36.2%	4.3%
		Percentage contribution of depletion to (real) aNNI growth per annum								
Angola		36.1%	-11.7%	-11.7%	-36.2%	-77.0%	-71.2%	-33.8%	-63.3%	88.8%
Azerbaijan		10.8%	28.5%	-3.3%	-6.4%	-36.5%	-56.5%	-45.6%	-33.5%	23.5%
Ecuador		3.4%	1.0%	-2.3%	-7.8%	-7.6%	-5.1%	-1.2%	-1.6%	10.3%
Kazakhstan		4.8%	-3.4%	-3.7%	-8.5%	-7.8%	-5.5%	-2.8%	-7.7%	6.8%
Norway		3.7%	0.1%	-2.0%	-4.1%	-4.9%	-2.0%	0.5%	-5.0%	9.0%
Russian Federation		2.6%	-0.8%	-3.9%	-6.5%	-6.9%	-4.0%	-1.7%	-4.8%	6.6%
Saudi Arabia		4.3%	0.8%	-6.6%	-8.0%	-13.4%	-4.1%	-0.1%	-14.4%	20.7%
Sudan		0.8%	-2.4%	6.6%	-3.8%	-3.7%	-4.0%	-8.7%	-4.8%	7.3%
Trinidad & Tobago		3.7%	-1.4%	-4.0%	-3.0%	-8.8%	-3.9%	2.2%	-5.1%	7.3%
Venezuela		2.7%	1.1%	0.8%	-6.3%	-5.6%	0.0%	2.0%	1.8%	2.4%

Source: authors' own calculations and adapted from BP (2011), World Bank (2011a), Gelb *et al*. (2012)

(ii) Measuring depletion and saving based on proven reserves plus expected probable resources

In our empirical discussion so far, we have looked at inferred discoveries only. We now turn to the total expected resource approach to wealth measurement as given by expression (24), and then derive measures of depletion and net saving based upon this revised wealth estimate. Critical to this is sufficient geological information not only on S but also X. [19] However, it is important to note that there is no definitive typology or even a uniform interpretation of each recognized classification of 'undiscovered' resources (for a discussion, see Rogner, 1997). The key point is that there is some probability that a given amount of resource could be added to proven reserves at some point in the future. What will determine these additions to reserves is the probability of technical and economic conditions shifting favorably over the passage of time or through learning or geological predictions being confirmed (or some combination of these developments).

The United States Geological Survey (USGS 2000, 2012) provides periodic assessments of undiscovered resources in the case of oil (see Rogner, 1997 and Mitchell, 2004).[20] What these USGS data describe is the extent of undiscovered resources along with given probabilities of those stocks being recoverable.[21] In Table 2 we use these data to show how the estimates in USGS (2012) on expected discoveries in regions of the world affect our accounting for the depletion of oil resources. The table values oil assets and depletion on the basis of proven resources (S) only and total resources ($S + X$).

Expression (24) indicates that in order to value to the *total* resource stock value ($S + X$), we require an estimate of discovery costs. In Table 2 we use an estimate of 5.0% of resource rent based on data on exploration and discovery costs for Norway's oil resources as reported in Annex Table A2. Since (in contrast to our theoretical model) resource discoveries are only loosely connected to discovery expenditures in any given year, our estimate of this value is based on the average cost per tonne of discovered oil from 1985 to 2010 (calculated as the mean value

[19] Rogner (1997) provides a good discussion of nuances in established definitions of what constitutes proven reserves for energy resources. These different definitions may result in distinct estimates of reserve extent with this difference largely explained by the specificity of readiness for economic exploitation in a definition. Moreover, Rogner along with a number of authors including Owen *et al*. (2010) and Mitchell (2004) all urge caution generally in the interpretation of available data on the extent of resources and reserves. This is not only because of the scientific and economic uncertainties but also the incentives that relevant agents have to report accurately these data.

[20] USGS (2000, 2012) indicates that this assessment is based primarily on geological assurance although some consideration is also given to technical and economic factors in determining extent. In using these probabilities, in this paper, we are in effect assuming that to evaluate these expected discoveries will be recoverable (i.e. added to proven reserves) with that likelihood. In practice, previously undiscovered resources might be added to proven resources or contingent resources (i.e. possible or probable reserves) with the latter being associated with some probability of ultimately being transferred to the former.

[21] Specifically, these probabilities are 95%, 50% and 5%. While not the complete probability distribution for (expected) undiscovered reserves, these data give some indication of the possible extent of resources at three points of the (truncated) lognormal distributions estimated in USGS (2000, 2012). While incomplete this provides at least some approximate basis with which to illustrate our approach to accounting for the case of expected probable reserves.

of cumulative discovery costs divided by the cumulative quantity of oil discovered over these years).

Table 2: Depletion values for oil adjusted for expected undiscovered resources, 2010

	Depletion as function of proven resource stock		***Depletion as function of total resource stock***	
	Depletion $m (% GNI)	**Resource stocks $m (*S*/*R*)**	**Depletion $m (% GNI)**	**Resource stocks $m ($S^*/R$)**
Former Soviet Union	$249,554	$5,571,338	$237,348	$6,004,281
	(13.8%)	(22.3)	(13.1%)	(25.3)
Middle East & North Africa	$197,736	$15,966,091	$179,175	$16,174,524
	(11.7%)	(80.7)	(10.6%)	(90.3)
Asia Pacific	$119,319	$1,674,577	$86,771	$2,850,715
	(0.8%)	(14.0)	(0.6%)	(32.9)
Europe	$70,833	$624,858	$62,864	$962,386
	(0.4%)	(8.8)	(0.4%)	(15.3)
North America (excl. USA)	$91,544	$1,775,696	$60,734	$2,784,314
	(3.6%)	(19.2)	(2.4%)	(45.8)
South & Central America	$43,179	$4,029,611	$31,501	$4,105,911
	(1.1%)	(93.3)	(0.8%)	(130.3)
Sub-Saharan Africa	$73,722	$2,424,395	$40,461	$3,169,246
	(6.9%)	(32.9)	(3.8%)	(78.3)
South Asia	$15,848	$388,789	$11,847	$511,206
	(0.7%)	(24.5)	(0.5%)	(43.2)

Source: authors' own calculations, USGS (2012), BP (2011), World Bank (2011a)

Table 2 describes our findings across regions. For Sub-Saharan Africa, the extent of undiscovered resources is such that the depletion value for this region falls by nearly half. In the case of Asia Pacific, the difference between depletion values is notable in dollar terms but less significant in terms of percentage of GNI. For most other regions, the differences in depletion value are less significant reflecting in part that there is less likelihood that future discoveries will be extensive relative to existing proven reserves (Gelb *et al*. 2012).

Table 2 also provides an indication of the asset value of S and S*. In some instances, the physical extent of expected discoveries might not be great (at least compared to proven resources) as is the case, for example, for the Former Soviet Union and Middle East & North Africa. However, for other regions, this physical stock of S* is considerably larger than for S alone. Examples here include Asia Pacific, Sub-Saharan Africa, South & Central America and North America (excl. USA). Table 2 indicates the magnitudes of these differences by showing the respective expected lifetimes of resource in parentheses in columns 2 and 4 (i.e. S/R and S*/R).

Whether these (expected) physical differences translate in substantially revised estimates of the value of oil resource stocks is another matter. This stock value for any region is the discounted value of a stream of (constant) future oil extraction. As a result, when proven reserve lifetimes are initially large, the impact on resource stock values of adding expected discoveries can be small. The contrasting cases here are Asia Pacific and South & Central America.

It would be useful to have these data on undiscovered resources for individual countries. Unfortunately, comparable country-level data are sparse. USGS (2000), however, provides a detailed country assessment and Table 3 presents results on alternative measures of depletion and genuine saving for the same 10 countries presented in Tables 1a and 1b.

For Azerbaijan, Kazakhstan, Saudi Arabia and Venezuela, the extent of proven reserves is relatively large. What this means is that the large corresponding (proven) reserves to production ratio has a significant bearing already on the value of resource depletion. In the case of Saudi Arabia, for example, for a proven reserve life of some 76 years, this oil depletion value is 14.8% of GNI. In the case of other countries in Table 3 such as Trinidad and Tobago and Russia, however, proven reserves are estimated to imply resource lifetimes roughly within one generation (from the year 2000). For example, Norway's proven reserve lifetime of 9 years translates in that case into an oil depletion value that is 14.0% of its GNI.

Table 3: Oil depletion values and genuine saving for selected countries, 2000

	GNI $ million	***Depletion as function of proven resource stock***			***Depletion as function of total resource stock***		
		Reserve life (S/R)	**Oil depletion (% GNI)**	**Genuine saving (% GNI)**	**Reserve life ($S+X/R$)**	**Oil depletion (% GNI)**	**Genuine saving (% GNI)**
Angola	$7,449	22	51.5%	-11.7%	62	28.6%	11.2%
Azerbaijan	$4,987	59	17.9%	-12.8%	116	9.9%	-4.8%
Ecuador	$14,530	31	13.7%	3.6%	35	12.8%	4.5%
Kazakhstan	$17,038	92	10.2%	-5.5%	150	6.4%	-1.7%
Norway	$166,018	9	14.0%	8.2%	20	11.5%	10.7%
Russia	$252,972	25	12.9%	15.8%	56	8.1%	20.6%
Saudi Arabia	$188,922	76	14.8%	5.3%	102	11.4%	8.7%
Sudan	$11,303	9	11.6%	-11.0%	26	8.5%	-8.0%
Trinidad & Tobago.	$7,526	17	11.2%	5.4%	37	8.0%	8.7%
Venezuela	$115,760	65	8.9%	14.4%	78	7.6%	15.7%

Notes: "Total reserves" refer to proven + expected reserves. These reserve life estimates are based on assessments of economic and technological feasibility of extraction. Given that these assessments (and the knowledge and assumptions on which they are based) are evolving, such estimates are also subject to change over time.

Source: Authors' calculations from USGS (2000), World Bank (2011a)

As expected, re-estimating reserve lives to include expected undiscovered resources shrinks the value of oil depletion in all cases. Clearly, the extent varies depending in large part on how much expected undiscovered resources add to total reserves. For many of these countries, however, the effect of considering this is to roughly double our estimate of total reserves. The largest absolute

changes are for Azerbaijan and Kazakhstan where expected undiscovered resources add respectively 57 and 58 years to total resource lifetimes. For Azerbaijan, the corresponding oil depletion as a percentage of GNI is 9.9% where depletion value is a function of total resources, compared to 17.9% where depletion value is a function of proven reserves only. This disparity is even more striking for Angola where the depletion value shrinks to 28.6% of its GNI once expected resources are taken in account.

Table 3 also indicates genuine saving rates for these 10 countries. Genuine saving is defined here as net saving (i.e. gross saving net of depreciation of produced capital) minus the value of oil depletion.[22] For Angola, Azerbaijan, Kazakhstan and Sudan, the genuine saving rate in 2000 is negative based on depletion values estimated as a function of proven oil reserves only. When we consider the additional role of expected discoveries in determining reserve life, the re-estimated genuine saving rate is higher. Indeed, for Angola, genuine saving becomes positive when depletion values are measured on the basis of total resources. We reflect further on the implications of such findings for thinking about fiscal prudence in our concluding section.

Conclusions

Our theoretical model of resource exploration with learning yields a number of useful insights. The expressions for saving and income derived in this model are formally the same as those derived for the Hartwick (1993) model, which was based upon Pindyck (1978). Discovery expenditures increase saving, with the amount discovered valued at the marginal discovery cost, while depletion reduces saving, with the amount depleted valued at the marginal rental rate. This result drives our empirical estimates of *aNNI* based on the 'marginalist' approach.

In our model, the fact that the ultimate stock of the resource is finite turns discovery into a process of depleting undiscovered resources. The result is that marginal discovery costs rise, driven by the Hotelling process seen in expression (15). In addition, the process of learning from extraction ties together the values of scarcity rents and cumulative knowledge as seen in expression (16). Because the model combines the effects of exhausting profitable discoveries and learning from resource extraction, the question of whether the total stock X will be discovered on the optimal path is in the end an empirical one, dependent on initial conditions and functional forms.

The fact that total resources $S + X$ are finite creates the risk of unsustainability on the optimal growth path, owing to the declining marginal product of capital and the constant pure rate of time preference. The generalized Hartwick Rule provides a policy option for resource-extracting countries concerned about sustainability. By ensuring that investment including the value of resource discovery is larger than depletion, this policy rule ensures that income and consumption will rise over time as the resource is exhausted.

Another useful insight from the theory concerns the wealth accounts. Here the model implies that we should treat the total resource stock $S + X$ (proven plus undiscovered resources) as an asset,

[22] Hence, these genuine saving estimates are deliberately conservative. These estimates do not consider, for example, the value of the depletion of gas resources in Trinidad and Tobago.

but we need to subtract the present value of future discovery costs in order to arrive at total wealth. As noted, however, it may not be optimal to discover all of the stock X.

When it comes to practical wealth accounting, we turn to Hicksian approaches to income measurement, although we modify this approach by excluding capital gains – setting aside issues of the distribution of income, capital gains cannot affect aggregate social welfare in a closed economy. We also explore the measurement of depletion and genuine saving when the total resource ($S + X^*$, where X^* is measured as the expected value of probable resources) is the basis for the accounting, thereby abstracting completely from year to year resource discovery.

Our empirical application of these approaches shows that (i) a strict marginalist approach to accounting for resource discoveries in selected oil-producing countries, as the formal model would suggest, leads to extreme volatility in measured income, amounting to several hundred percent of *aNNI* in some cases; (ii) when our modified Hicksian approach to income measurement is employed, the volatility is substantially reduced, and the contribution of discoveries and depletion to this volatility is reduced as well; and (iii) when depletion and genuine saving are based upon the total resource (rather than proven reserves) in selected oil producing countries, there is a substantial reduction in the measured value of depletion and an increase in genuine saving as a result.

Whether the modified Hicksian approach to accounting for discoveries or accounting for the total expected resource is employed, one result is that many countries have apparent resource endowments measured in decades. This observation leads us back to issues of fiscal prudence.

As we noted, the question of fiscal prudence is fundamentally about how much to consume and how much to save out of current resource rents, assuming that resource taxes capture a substantial proportion of these rents. For a resource exporter Hamilton and Bolt (2004) show that capital gains on future exports and capital gains on future financial assets should also be included in genuine saving. In addition, van der Ploeg (2012) introduces exogenous technological change in the resource extraction cost function, and shows that net saving must also include a capital gain in the form of the present value of increases or decreases in future extraction costs. Prudent fiscal policies for exhaustible resource exporting economies would need to take all of these factors into account.

As Figure 1 shows, the proportion of resource rents that should be saved in order to maintain real wealth declines monotonically with increasing resource lifetimes. However, Figure 1 assumes a constant quantity of extraction and constant unit value of resource rents over the whole resource lifetime. Given high resource price volatility, uncertainties about future technologies and substitution possibilities, and policy uncertainties linked to phenomena such as climate change, this is a heroic assumption. The alternative would be to forecast unit rents and quantities extracted, and then to calculate the depletion shares of total rent numerically. For exporters, forecasts of capital gains would also be required.

The cost of fiscal policy mistakes could be substantial for many countries. If the major portion of resource rent is consumed in the short run, based on forecasts of buoyant long run revenue and capital gains, but major downside risks are actually realized, the country may find that it has

consumed a large proportion of its wealth. Countries may wish to hedge against downside risks, and one way to do this would be to use conservative forecasts of unit rents, resource extent, quantities extracted and capital gains. Countries would no doubt vary in their degree of prudence, driven in part by discount rates and their assumptions about the size and likelihood of downside risks in exhaustible resource markets.

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Annex 1: The Generalized Hartwick Rule and Genuine Saving

We derive the generalized Hartwick rule and an expression which indicates the welfare significance of genuine saving.

From expression (6) we have,

$$\ddot{K} = F_K\dot{K} + F_R\dot{R} - \dot{C} - v_D\dot{D} - v_Z\dot{Z} - v_Q\dot{Q} \tag{A1}$$

Taking the time derivative of genuine saving G (expression 11), and applying expressions (A1), (14) and (15) in succession we derive,

$$\begin{aligned} \dot{G} &= F_K\dot{K} + F_R\dot{R} - \dot{C} - v_D\dot{D} - v_Z\dot{Z} - v_Q\dot{Q} - \dot{F}_R R - F_R\dot{R} + \dot{v}_D D + v_D\dot{D} \\ &= F_K\dot{K} - \dot{C} - v_Z R - v_Q D - F_K F_R R + v_Z R + v_Q D + F_K v_D D \\ &= F_K G - \dot{C} \end{aligned}$$

Rearranging terms we have the generalized Hartwick rule formula,

$$\dot{C} = F_K G - \dot{G} \tag{A2}$$

Applying the Ramsey formula (expression 13) to this expression, and noting that $\dot{U} = U_C\dot{C}$ we get,

$$\dot{U} = \rho U_C G - \frac{d}{dt} U_C G \tag{A3}$$

This is the generalized Hartwick rule formula with saving measured in utils rather than dollars.

Now note that the Hamiltonian function for the model can be written as,

$$H = U + U_C G \tag{A4}$$

Combining this with expression (A3) we derive,

$$\dot{H} = \rho U_C G \tag{A5}$$

Turning to social welfare V, it follows from expressions (1) and (A4) that,

$$\dot{V} = \rho V - U = \rho V - H + U_C G,$$

and therefore that,

$$\rho\dot{V} = \rho^2 V - \rho H + \rho U_C G = \rho^2 V - \rho H + \dot{H}$$

This expression has particular solution,

$$H = \rho V \tag{A6}$$

From this expression and expression (A5) we therefore conclude that,

$$G = \frac{1}{U_C}\dot{V} \tag{A7}$$

Genuine saving is therefore equal to the dollar-valued change in social welfare.

Table A.1: Imputed Discoveries for Selected Oil Producing Countries, 2000 to 2009

		2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Angola	*Million tonnes*	163	108	371	30	81	62	70	694	95	90
	% proven reserves	20.0%	12.2%	30.6%	2.5%	6.6%	5.1%	5.7%	37.7%	5.2%	4.9%
	Implied additional reserve life (years)	4.4	3.0	8.4	0.7	1.6	1.0	1.0	8.2	1.0	1.0
Azerbaijan	*Million tonnes*	14	15	809	15	15	22	32	41	42	48
	% proven reserves	8.6%	9.2%	84.8%	1.6%	1.6%	2.3%	3.4%	4.3%	4.4%	5.1%
	Implied additional reserve life (years)	1.0	1.0	53.2	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Ecuador	*Million tonnes*	39.2	29.4	78.6	21.3	26.6	0.4	-27.6	-37.4	368.9	-0.6
	% proven reserves	6.5%	4.7%	12.4%	3.1%	3.9%	0.1%	-4.2%	-6.1%	67.6%	-0.1%
	Implied additional reserve life (years)	2.1	1.4	3.8	1.1	1.3	0.0	-1.0	-1.4	14.2	0.0
Kazakhstan	*Million tonnes*	37.0	2033.1	50.7	55.3	64.6	95.9	71.0	73.9	77.4	84.0
	% proven reserves	1.1%	59.6%	0.9%	1.0%	1.2%	1.8%	1.3%	1.4%	1.4%	1.5%
	Implied additional reserve life (years)	1.2	54.9	1.2	1.1	1.2	1.5	1.1	1.0	1.0	1.1
Norway	*Million tonnes*	225.0	201.6	9.2	121.7	100.5	143.2	-17.5	75.5	30.0	61.0
	% proven reserves	15.1%	13.0%	0.6%	8.5%	7.3%	10.8%	-1.3%	6.5%	2.7%	6.0%
	Implied additional reserve life (years)	1.4	1.2	0.1	0.7	0.6	0.9	-0.1	0.5	0.2	0.5
Russian Federation	*Million tonnes*	383.0	962.2	1657.5	737.9	400.8	580.3	65.9	576.7	899.2	595.2
	% proven reserves	4.8%	11.9%	19.1%	7.4%	3.9%	5.7%	0.6%	5.8%	9.0%	5.7%
	Implied additional reserve life (years)	1.2	3.0	4.7	1.9	0.9	1.3	0.1	1.2	1.8	1.2
Saudi Arabia	*Million tonnes*	470.1	449.1	457.2	497.8	745.1	539.8	545.8	514.5	520.1	564.4
	% proven reserves	1.3%	1.3%	1.3%	1.4%	2.1%	1.5%	1.5%	1.4%	1.4%	1.6%
	Implied additional reserve life (years)	1.1	1.0	1.0	1.1	1.5	1.0	1.0	1.0	1.0	1.0
Sudan	*Million tonnes*	55	24	12	779	28	15	46	36	23	24
	% proven reserves	67.1%	25.2%	12.6%	90.5%	3.2%	1.7%	5.1%	3.9%	2.5%	2.6%
	Implied additional reserve life (years)	6.2	2.3	1.0	58.8	1.8	1.0	2.7	1.5	1.0	1.0
Trinidad & Tobago	*Million tonnes*	11.0	21.6	30.2	-23.8	-3.6	7.7	7.4	18.2	1.8	7.5
	% proven reserves	9.8%	18.6%	23.1%	-15.5%	-3.0%	7.0%	6.7%	16.8%	1.5%	6.6%
	Implied additional reserve life (years)	1.6	3.1	4.5	-3.1	-0.4	1.0	0.9	2.1	0.2	1.0
Venezuela	*Million tonnes*	161.3	270.6	92.6	116.1	486.1	184.8	1137.2	1774.1	10077.2	5420.5
	% proven reserves	1.5%	2.6%	0.9%	1.1%	4.6%	1.7%	10.4%	14.9%	74.3%	23.1%
	Implied additional reserve life (years)	1.0	1.7	0.6	0.8	3.8	1.3	7.8	12.7	77.5	42.6

Source: authors' own calculations and adapted from BP (2011), Gelb *et al*. (2012)

Table A.2: Discovery costs for Norway's oil resources

Year	Cumul Costs (million 2010 USD	Cumul discoveries (mt. 1985=0)	Cost per tonne oil discovered (2010 USD)	Oil Price (2010 USD)	Norway unit cost (2010 USD)	Net rent (2010 USD)	Discovery cost as % net rent
1985	2,697	553	4.9	360.7	65.7	285.1	0.02
1986	4,862	849	5.7	186.3	60.3	116.9	0.05
1987	6,325	1,045	6.1	228.8	53.7	167.0	0.04
1988	7,478	1,132	6.6	179.3	53.4	117.9	0.06
1989	8,805	1,170	7.5	209.5	52.0	149.7	0.05
1990	10,113	1,227	8.2	258.9	48.1	203.5	0.04
1991	12,117	1,446	8.4	212.0	47.6	157.2	0.05
1992	13,964	1,660	8.4	203.8	49.3	147.2	0.06
1993	15,242	1,672	9.1	176.6	45.3	124.5	0.07
1994	16,404	1,893	8.7	163.3	43.1	113.7	0.08
1995	17,466	1,939	9.0	172.6	42.8	123.3	0.07
1996	18,698	2,031	9.2	201.5	42.8	152.3	0.06
1997	20,512	2,587	7.9	185.6	41.9	137.5	0.06
1998	22,139	2,695	8.2	124.9	45.4	72.7	0.11
1999	23,213	2,721	8.5	170.2	53.7	108.5	0.08
2000	24,243	2,839	8.5	260.3	50.2	202.5	0.04
2001	25,555	2,884	8.9	219.5	53.0	158.6	0.06
2002	26,349	2,896	9.1	221.1	54.1	158.9	0.06
2003	27,114	3,001	9.0	250.9	54.6	188.1	0.05
2004	27,852	3,019	9.2	318.6	56.6	253.6	0.04
2005	29,144	3,092	9.4	436.3	63.6	363.2	0.03
2006	31,239	3,094	10.1	508.8	71.1	427.0	0.02
2007	34,205	3,148	10.9	546.7	79.2	455.7	0.02
2008	38,241	3,316	11.5	729.8	99.0	616.0	0.02
2009	42,864	3,456	12.4	456.3	107.4	332.8	0.04
2010	46,948	3,600	13.0	579.4	120.2	441.1	0.03

Source: adapted from NPD (2011)