Term structures of discount rates: An international perspective

Christian Gollier
TSE and UCL
—
Olivier Mahul
The World Bank

November 22, 2017

Motivation

- The choice of the discount rate is critical for long-dated investment decision-making.
- Discounting theory remains a dog's breakfast:
  - Misleading dogma for a single discount rate;
  - Some experts recommend a small risk-free discount rate, and others a large risk-adjusted one;
  - Theory (Ramsey, CCAPM) and Markets disagree (financial puzzles);
  - Debate about the term structure (gamma discounting, risk-free/risky);
  - Debate about an ecological discount rate (relative prices);
  - Total separation between financial economics and environmental economics.
- No consensus $\Longrightarrow$ no "Law of a single price" $\Longrightarrow$ inefficient investments!

Aversion to intertemporal inequality ρ

- In a growing economy, investing for the future raises intertemporal inequalities of consumption.
- Under intertemporal inequality aversion, it reduces intertemporal welfare.
- In a risk-free world, the discount rate net of the rate of impatience δ is the IRR of a safe project that compensates for this adverse effect.
- Ramsey (1928) tells us that this is the product of the relative aversion to inequality ρ by the anticipated growth rate of consumption:

$$\text{discount rate} = \delta + (\rho \times \text{growth rate of consumption}).$$

- Observing how the short interest rate change with growth, Hansen and Singleton (1983) and Hall (1985) suggested $\rho \simeq 1$.
- Precautionary reduction of the risk-free rate because the future is uncertain.

Aversion to risk γ

- Because of risk aversion, projects that raise the aggregate risk should be penalized.
- This can be done by adjusting the discount rate upwards by a risk premium.
- In the Discounted Expected Utility (DEU) model, the project-specific risk premium equals

$$\text{risk premium} = \phi \times \gamma \times \text{variance of growth rate},$$

  where ϕ is the CCAPM beta of the project.
- In the U.S., the CCAPM beta of equity is around $\phi \simeq 3$, the equity premium is around 6%, and the variance of growth is around $(3\%)^2 \simeq 0.1\%$.
- Thus, risk aversion γ should be around $\gamma \simeq 20$.
- But in the DEU model, ρ equals γ. $\Longrightarrow$ Financial puzzles.

Measuring risk aversion

risk aversion	certainty equivalent
0.5	0.93
0.67	0.91
1	0.87
2	0.75
10	0.54
20	0.52

Table: Certainty equivalent consumption of a 50-50 chance of consuming either 0.5 forever, or 1.5 forever, as a function of relative risk aversion.

Recursive preferences, persistence, and term structures

- The Epstein-Zin-Weil recursive utility model allows us to disentangle ρ from γ.
- But with iid shocks, the formula for the risk premium remains unchanged, and $\gamma = 20$ is definitely unrealistic (Weil (1989)).
- In the long-run risks literature initiated by Bansal and Yaron (2004), a new ingredient is combined with EZW preferences:
  - The persistence of shocks to the consumption growth rate.
- Because it magnifies future risks, it makes the term structure of the risk-free rates and risk premia respectively decreasing and increasing (Gollier (2012)).
- In the DEU model, it does not affect short-term rates. But in the EZW model, it reduces the short risk-free rate and it raises the short risk premium.

What's new in this paper?

- We characterize analytically the term structures of social risk-free discount rate and social risk premia when the representative agent
  - has Epstein-Zin-Weil preferences;
  - believes that the growth rate of consumption follows an AR(1) process.
- This is new and simpler compared to the existing long-run risks literature.
- As an important departure from the finance literature which focuses on the U.S. and the Western world, we calibrate the model to 248 countries and economic zones.
- Our results illustrates the great diversity of growth processes around the world, implying important differences in the way investment projects should be evaluated.

The model

- Social preferences: Epstein-Zin-Weil

$$
\begin{aligned}
V_t^{1-\rho} &= (1-\beta)c_t^{1-\rho} + \beta\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\rho}{1-\gamma}} && \text{if } \rho \neq 1 \\
\log V_t &= (1-\beta)\log c_t + \beta \log\left(E_t V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} && \text{if } \rho = 1.
\end{aligned}
$$

- Growth process: AR(1)

$$x_t = \log(c_t/c_{t-1})$$

$$x_{t+1} = \mu + k(x_t - \mu) + \sigma\eta_{t+1}, \tag{1}$$

- We are interested in valuing at date 0 an action that generates a single marginal payoff εc_t^{ϕ} at date t, for some $\phi \in \mathbb{R}$.
- Such a marginal investment has an impact on V_0 equivalent to $\varepsilon E_0 c_t^{\phi}$ discounted at rate

$$r_{0,t}^f + \pi_{0,t}(\phi).$$

Parameters and variables

Parameters	Interpretation
ρ	Relative aversion to intertemporal inequality
γ	Relative aversion to risk
β	Factor of pure preference for the present
δ	Rate of pure preference for the present
μ	Trend of consumption growth
σ	Volatility of conditional consumption growth
k	Persistence coefficient
ϕ	Consumption-based CAPM beta of the project

Variables	Interpretation
$r^f_{0,t}$	Social risk-free discount rate for maturity t
$\pi_{0,t}(\phi)$	Risk premium of investment ϕ for maturity t

Term structure of the annualized variance of growth rate

- Let $x_{0,t} = \log(c_t/c_0)$ be the change in log consumption between 0 and t.

$$\frac{E_0[x_{0,t}]}{t} = \mu + (x_0 - \mu)k\frac{1-k^t}{1-k}$$

$$\frac{Var_0[x_{0,t}]}{t} = \frac{\sigma^2}{(1-k)^2}\left(1 - 2\frac{k-k^{t+1}}{t(1-k)} + \frac{k^2-k^{2t+2}}{t(1-k^2)}\right)$$

- On average, $x_0 = \mu$ and the term structure of the annualized expectation of the growth rate is flat.
- The term structure of the annualized variance of the growth rate is
  - flat when $k = 0$;
  - increasing when k is positive.

Characterization of term structures

$$b = (1+b)\beta k \exp\left((1-\rho)\left(\mu + \frac{1}{2}(1-\gamma)(1+b)\sigma^2\right)\right).$$

Proposition

The interest rates and risk premia can be approximated as follows:

$$\begin{aligned} r^f_{0,t} = \delta \quad &+ \quad \rho\frac{E_0[x_{0,t}]}{t} - \frac{\rho^2}{2}\frac{Var_0[x_{0,t}]}{t} - \frac{(1-\rho)(\gamma-\rho)(1+b)^2}{2}\sigma^2 \\ &- \quad \frac{\rho(\gamma-\rho)(1+b)}{1-k}\left(1 - \frac{k-k^{t+1}}{t(1-k)}\right)\sigma^2, \end{aligned}$$

and $\pi_{0,t}(\phi) = \phi\pi_{0,t}(1)$ *with*

$$\pi_{0,t}(1) = \rho\frac{Var_0[x_{0,t}]}{t} + \frac{(\gamma-\rho)(1+b)}{1-k}\left(1 - \frac{k-k^{t+1}}{t(1-k)}\right)\sigma^2.$$

These approximations are exact when ρ *equals 1 or* γ*.*

Special case 1: Discounted Expected Utility ($\rho = \gamma$) and Geometric Brownian motion ($k = 0$)

- Risk-free rates: Extended Ramsey rule.

$$r^f_{0,t} = \delta + \rho\left(\mu + \frac{\sigma^2}{2}\right) - \gamma(1+\rho)\frac{\sigma^2}{2}$$

- Risk premia: CCAPM.

$$\pi_{0,t}(\phi) = \phi\gamma\sigma^2.$$

Special case 2: Discounted expected utility ($\rho = \gamma$) with persistence ($k > 0$)

- The persistence of shocks to growth magnifies the long-run risk.
- This makes the term structure of risk-free discount rates decreasing, on average:

$$r^f_{0,t} = \delta + \gamma \frac{\log\left(E_0 c_t\right)}{t} - \frac{\gamma}{2}(1+\gamma)\frac{Var_0[x_{0,t}]}{t}$$

- This also makes the term structure of aggregate risk premia increasing:

$$\pi_{0,t}(\phi) = \phi\gamma \frac{Var_0[x_{0,t}]}{t}.$$

- The persistence has no effect on the short interest rate and the short risk premium.

Comparative statics in the EZW model, $k \in [0, 1]$

Proposition

Suppose that γ is larger than ρ, and that ρ is smaller than 1. Then, the persistence of shocks to growth reduces the short interest rate $r^f_{0,1}$.

Proposition

Suppose that γ is larger than ρ. Then, the persistence of shocks to growth raises the short systematic risk premium $\pi_{0,1}(1)$.

Proposition

Suppose that γ is larger than ρ. Then, the term structures of the mean interest rates $Er^f_{0,t}$ and of the aggregate risk premia $\pi_{0,t}(1)$ are respectively decreasing and increasing.

Calibration

- For each of the 248 countries and economic zones contained in the World Bank database containing annual real growth rates of GDP/cap for the period 1961-2015, we estimate the triplet (μ, σ, k).
- Following classical estimates in the long-run risks literature, we assume
  - A rate of pure preference for the present of $\delta = 1\%$;
  - A relative aversion to intertemporal inequality of $\rho = 2/3$;
  - A relative aversion to risk of $\gamma = 10$.

Calibration: USA

Country	μ	σ	k	$\overline{r}^f_{0,1}$	$\overline{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$
China	7.48	4.37	0.37	3.27	2.17	2.99	4.74
European Union	2.25	1.54	0.48	2.08	1.83	0.44	0.83
France	2.11	1.55	0.57	1.85	1.43	0.53	1.20
Latin America	1.73	2.10	0.40	1.49	1.20	0.72	1.19
ME & North Africa	1.76	3.20	0.46	0.43	-0.54	1.83	3.37
Nicaragua	0.47	5.49	0.36	-2.76	-4.32	4.53	7.00
Sub-Saharan Africa	0.86	2.42	0.49	0.52	-0.11	1.09	2.08
United Kingdom	2.01	1.92	0.37	1.83	1.62	0.57	0.89
United States	2.08	1.89	0.31	1.94	1.80	0.51	0.73
World	1.85	1.35	0.37	1.98	1.88	0.28	0.45
Zimbabwe	0.02	6.08	0.40	-4.40	-6.82	5.91	9.75

Calibration: USA vs China

Country	μ	σ	k	$\overline{r}^f_{0,1}$	$\overline{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$
China	7.48	4.37	0.37	3.27	2.17	2.99	4.74
European Union	2.25	1.54	0.48	2.08	1.83	0.44	0.83
France	2.11	1.55	0.57	1.85	1.43	0.53	1.20
Latin America	1.73	2.10	0.40	1.49	1.20	0.72	1.19
ME & North Africa	1.76	3.20	0.46	0.43	-0.54	1.83	3.37
Nicaragua	0.47	5.49	0.36	-2.76	-4.32	4.53	7.00
Sub-Saharan Africa	0.86	2.42	0.49	0.52	-0.11	1.09	2.08
United Kingdom	2.01	1.92	0.37	1.83	1.62	0.57	0.89
United States	2.08	1.89	0.31	1.94	1.80	0.51	0.73
World	1.85	1.35	0.37	1.98	1.88	0.28	0.45
Zimbabwe	0.02	6.08	0.40	-4.40	-6.82	5.91	9.75

Calibration: USA vs Nicaragua

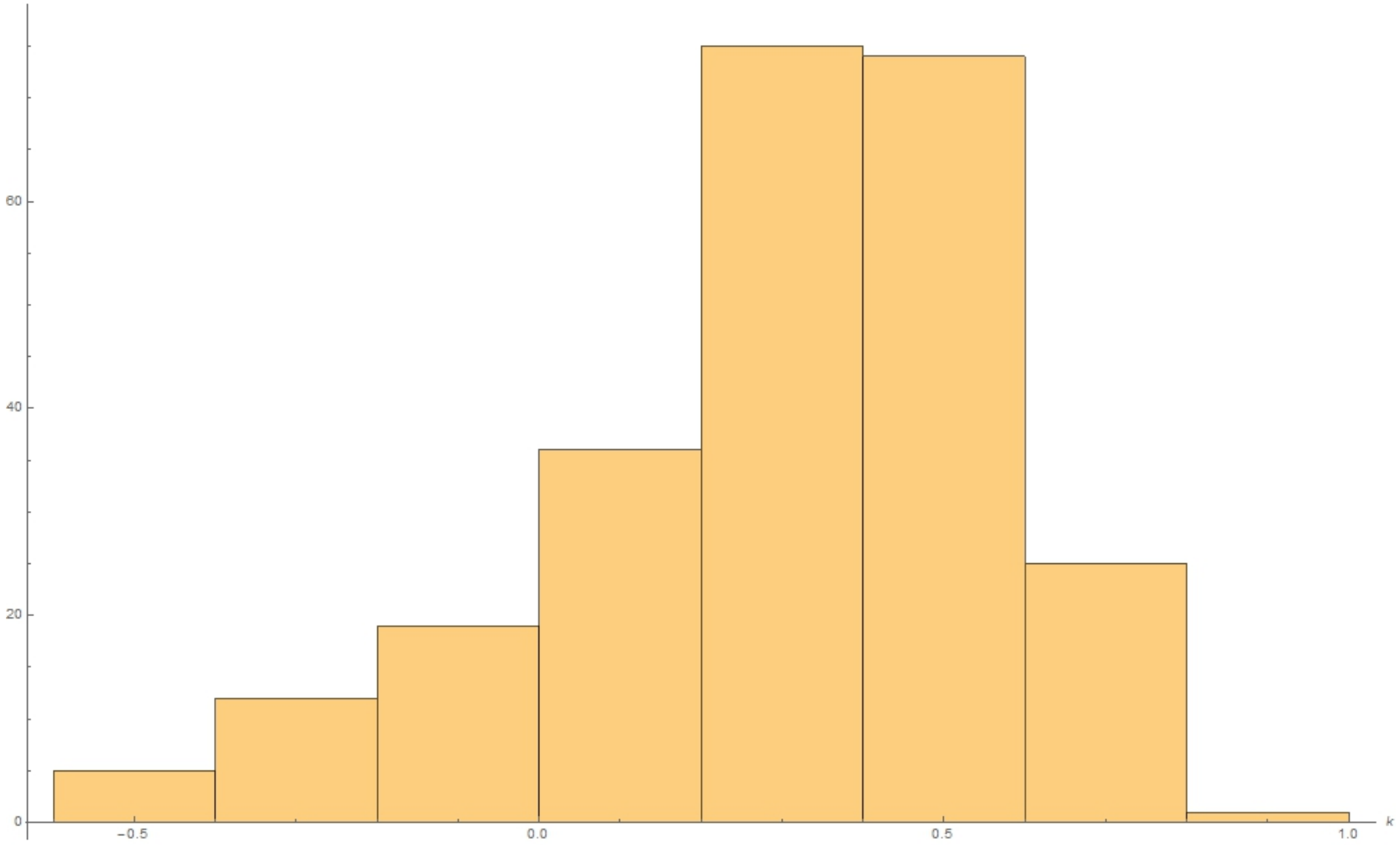
Country	μ	σ	k	$\overline{r}^f_{0,1}$	$\overline{r}^f_{0,20}$	$\pi_{0,1}$	$\pi_{0,20}$
China	7.48	4.37	0.37	3.27	2.17	2.99	4.74
European Union	2.25	1.54	0.48	2.08	1.83	0.44	0.83
France	2.11	1.55	0.57	1.85	1.43	0.53	1.20
Latin America	1.73	2.10	0.40	1.49	1.20	0.72	1.19
ME & North Africa	1.76	3.20	0.46	0.43	-0.54	1.83	3.37
Nicaragua	0.47	5.49	0.36	-2.76	-4.32	4.53	7.00
Sub-Saharan Africa	0.86	2.42	0.49	0.52	-0.11	1.09	2.08
United Kingdom	2.01	1.92	0.37	1.83	1.62	0.57	0.89
United States	2.08	1.89	0.31	1.94	1.80	0.51	0.73
World	1.85	1.35	0.37	1.98	1.88	0.28	0.45
Zimbabwe	0.02	6.08	0.40	-4.40	-6.82	5.91	9.75

Calibration: Summary statistics

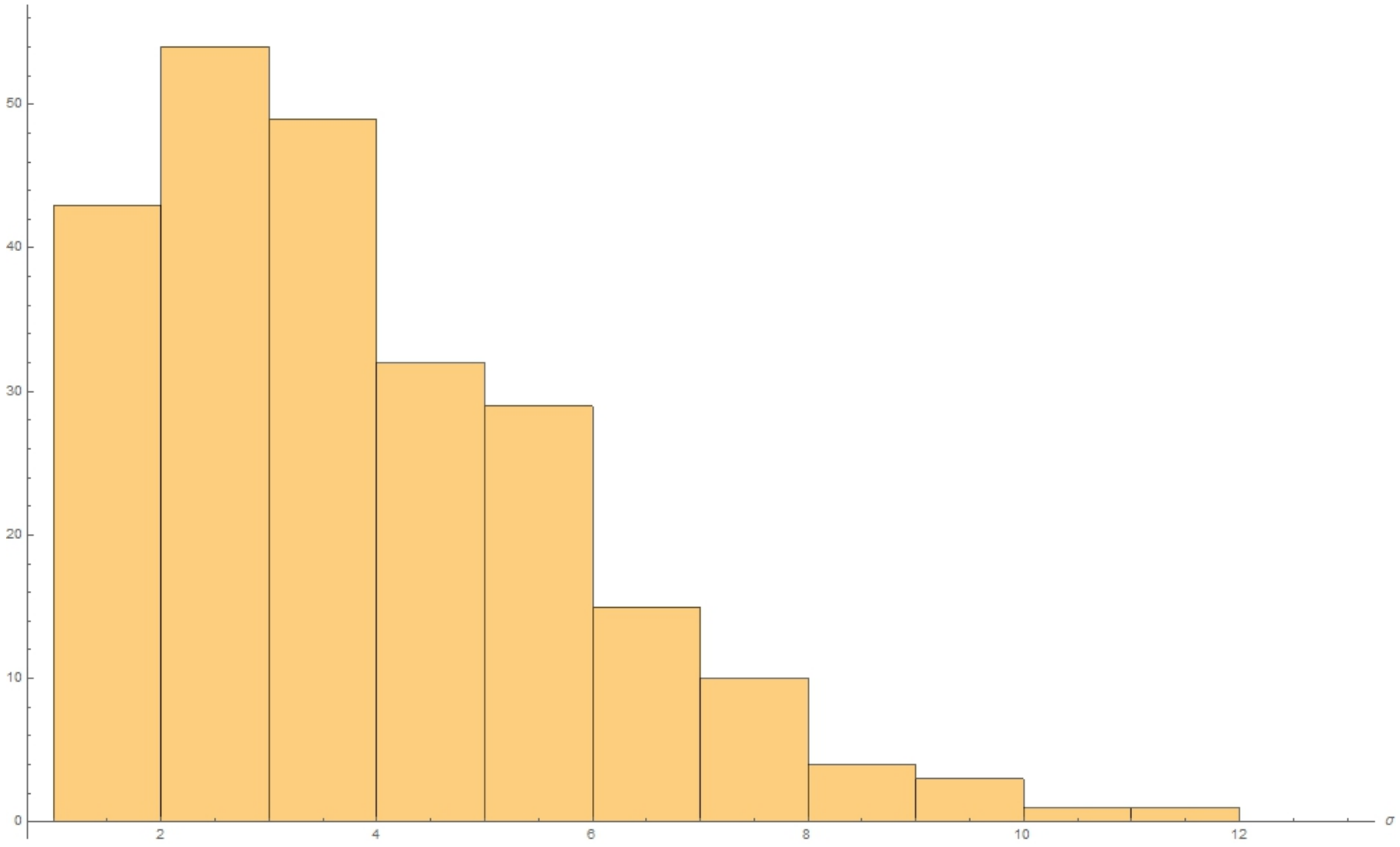
variable	mean	stdev	5%	50%	95%
μ	2.20	1.75	-0.31	2.07	5.40
σ	4.31	3.47	1.54	3.47	8.83
k	0.30	0.27	-0.22	0.33	0.69
$\overline{r}^f_{0,1}$	-1.42	8.72	-8.26	0.71	2.81
$\overline{r}^f_{0,20}$	-3.27	13.70	-16.30	0.14	2.62
$\pi_{0,1}$	4.21	10.10	0.43	1.79	11.80
$\pi_{0,20}$	7.12	17.50	0.57	2.56	22.90

Table: Summary statistics for the 248 countries and economic zones of the World Bank database. All rates are in percent.

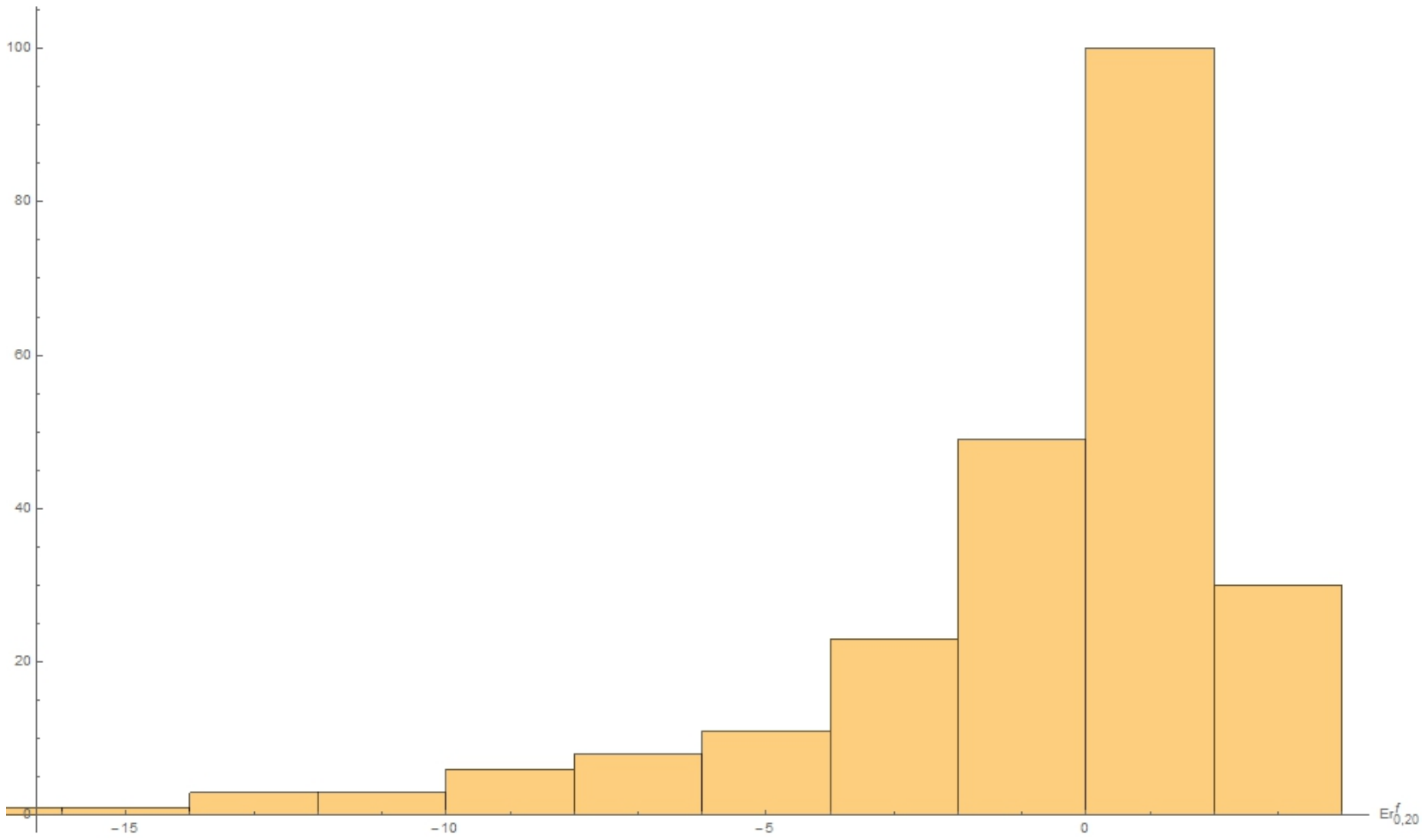
Histogram of k



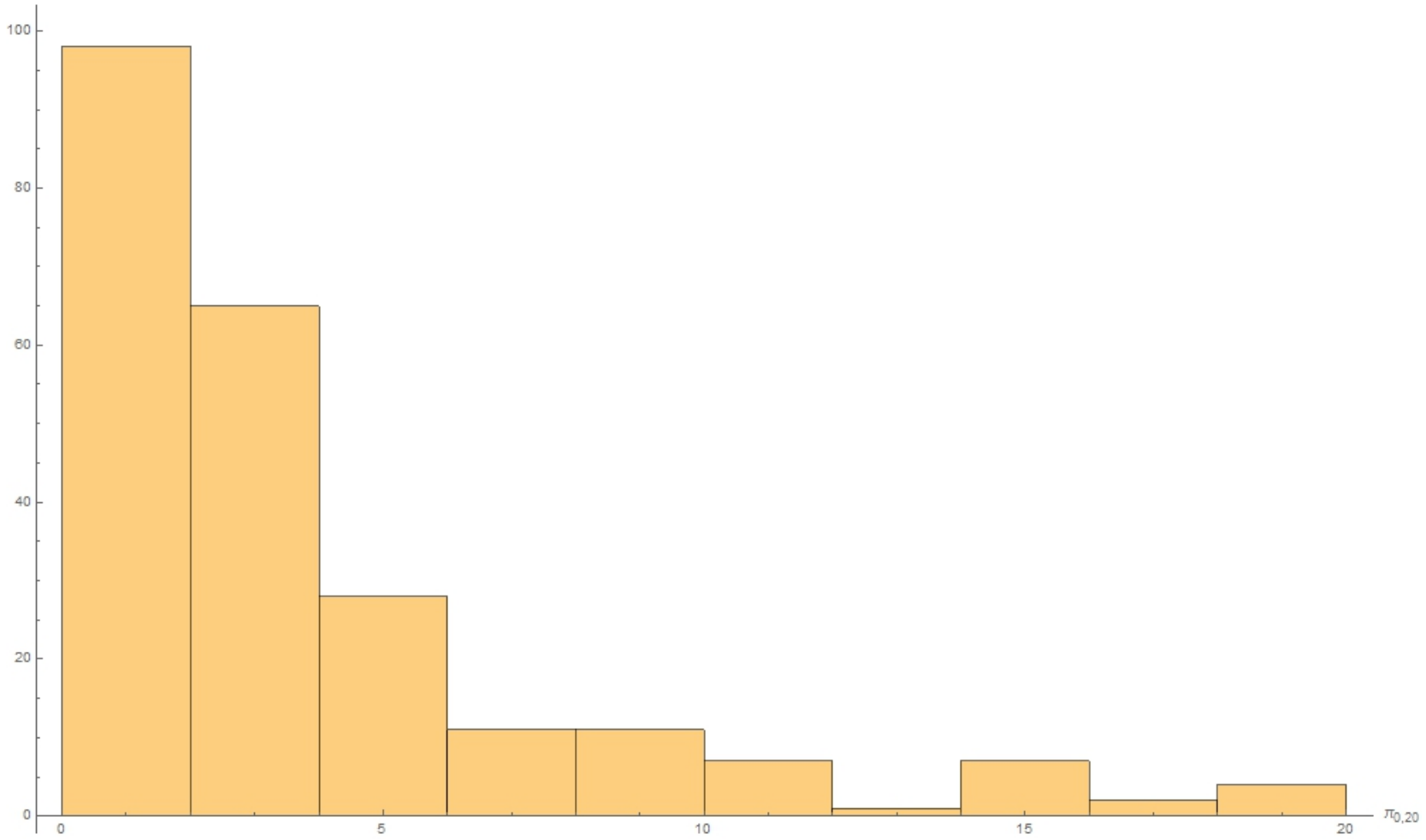
Histogram of σ



Histogram of $r^f_{0,20}$



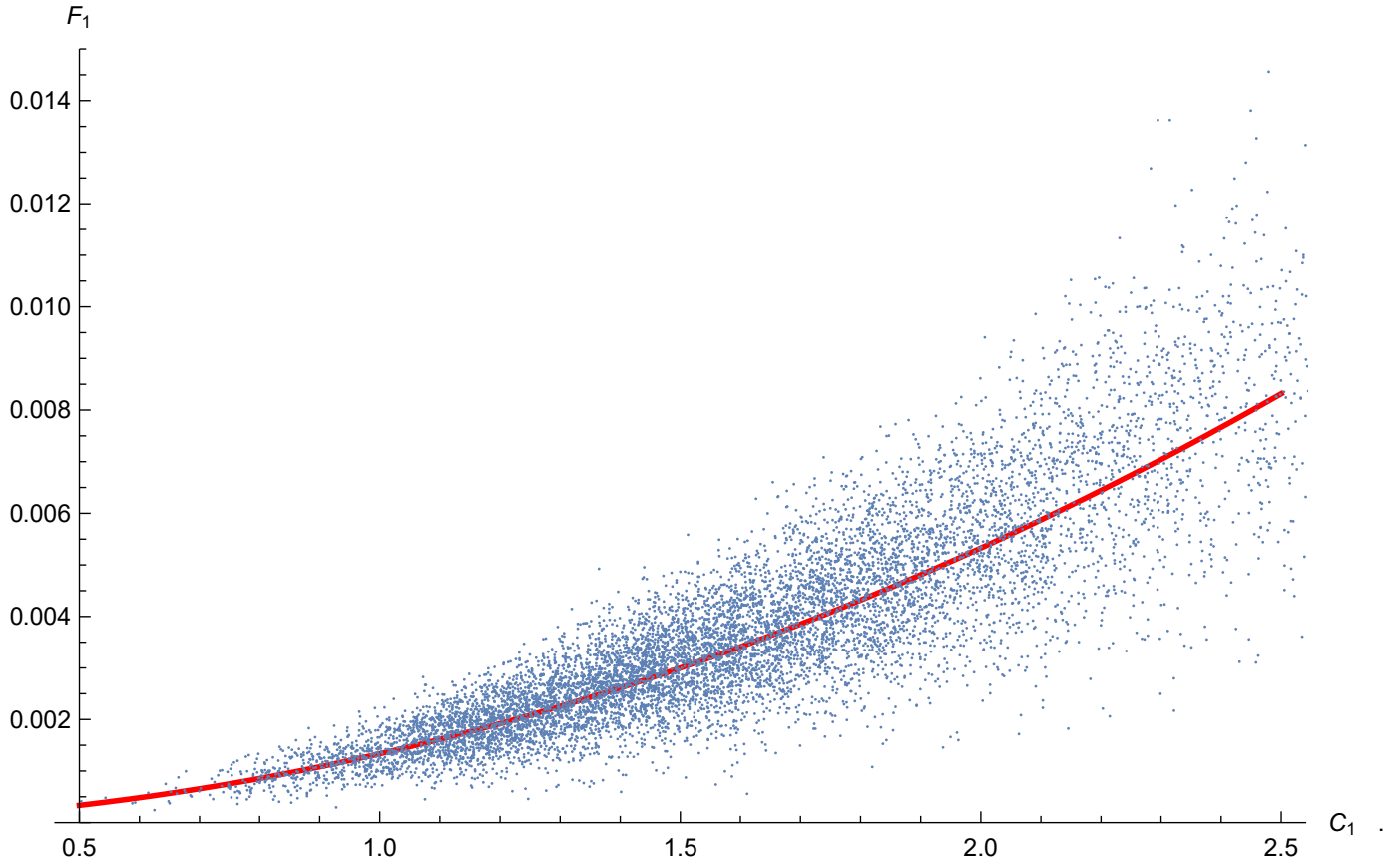
Histogram of $\pi_{0,20}(1)$



Climate beta

- What is the beta of investments whose aim is to reduce emission of CO_2?
- Two opposite stories:
  - Positive beta: A growth rate larger than expected raises CO_2 concentration and the marginal damage. There is a positive correlation between future consumption and the future benefit of mitigation.
  - Negative beta (Daniel, Litterman and Wagner (2015)): A larger climate sensitivity raises the marginal damages and reduces consumption.
- Dietz, Gollier and Kessler (2015): $\beta \simeq 1$.
- Because there is a consensus for a normative γ around 2, the term structure of the risk-adjusted discount rate for climate change should be flat ($\beta \simeq \gamma/2$).

Monte-Carlo simulation of DICE



Concluding remarks

- When determining how to discount the future, one should recognize that individuals are more averse to risk than to intertemporal inequalities.
- This reduces risk-free rates (possibly making them negative), and it raises risk premia.
- However, for long-dated investment projects, the sacrifices are for the current generations, and the benefits are for the future generation. In that case, ρ represents the aversion to inter*generational* inequalities.
- Under the veil of ignorance, the aversion to risk *is* the aversion to intergenerational inequalities.
- Our model is useful only for intragenerational projects.

Bansal, R. and A. Yaron (2004): "Risks For the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

Gollier, C. (2012): *Pricing the Planet's Future: The Economics of Discounting in an Uncertain World*, Princeton University Press.

Hall, R. E. (1985): "Real interest and consumption," NBER Working Paper 1694.

Hansen, L. and K. Singleton (1983): "Stochastic consumption, risk aversion, and the temporal behavior of asset returns," *Journal of Political Economy*, 91, 249–265.

Ramsey, F. (1928): "A mathematical theory of savings," *The Economic Journal*, 38, 543–559.

Weil, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24, 401–421.